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GL-TR-90-0282

Some Remarks on Compliance Testing

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September 1990

Scientific Report No. 3

Approved for public release; distribution unlimited

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AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AIR FORCE BASE, MASSACHUSETTS 01731-5000

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Defense Advanced Research Projects Agency
Nuclear Monitoring Research Office
ARPA ORDER NO. 5299

MONITORED BY
Geophysics Laboratory
F19628-88-K-0042

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704 0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 1990	Scientific #3

4. TITLE AND SUBTITLE
Some Remarks on Compliance Testing

5. FUNDING NUMBERS
PE 62714E
PR 8A10 TA DA WU AH
Contract F19628-88-K-0042

6. AUTHOR(S)
H. L. Gray
Wayne A. Woodward

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Southern Methodist University
Department of Statistical Science
Dallas, TX 75275

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Geophysics Laboratory
Hanscom AFB, Massachusetts 01731-5000

Contract Manager: James Lewkowicz/LWH

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
GL-TR-90-0282

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

We discuss several issues concerning compliance testing. First we discuss the F-number and provide an alternative definition which is identical to the common definition when $\sigma_{\hat{W}}$ is known but which is applicable to a broader range of testing situations. We discuss tests for compliance of a single new event given a collection of events for which we have both CORRTEX and seismic readings. These tests are considered in the case which σ_{sei} is assumed to be known and in the case in which σ_{sei} is unknown but the ratio $\sigma_{sei}/\sigma_{cor}$ is known. We examine the robustness of these tests under situations for which we do not have perfect knowledge of σ_{sei} or the ratio and show that the test based on the ratio is more robust to errors in specification of the variances. We also discuss the estimation of σ_{sei} based on the events for which we have both CORRTEX and seismic data. A brief discussion of the use of the mixture-of-normals model for purposes of estimating the seismic variance is also given.

14. SUBJECT TERMS
threshold test ban treaty, seismic monitoring, F-number, CORRTEX, compliance testing, robustness

15. NUMBER OF PAGES
58

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

SOME REMARKS ON COMPLIANCE TESTING

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Section 1

A Discussion of the F-Number

1.1 Introduction

The standard measure of the performance of a statistical test for compliance to the TTBT has become, to most, the so called F-number. The F-number is commonly defined by the simple expression

$$F(\sigma_{\hat{W}}) = 10^{1.96\sigma_{\hat{W}}} \tag{1.1}$$

where $\sigma_{\hat{W}}$ is the standard deviation of the estimated log-yield, where log-yields are assumed to be normally distributed. The motivation for such a definition is as follows. If $W = \log Y$, where $Y =$ yield of a given event and $\hat{W}$ is an estimate of W which is distributed $N(W, \sigma_{\hat{W}})$, then

$$P[W < \hat{W} + Z_\lambda \sigma_{\hat{W}}] = 1 - \lambda \tag{1.2}$$

and

$$P\left[10^{W} < 10^{\hat{W}} \cdot 10^{Z_\lambda \sigma_{\hat{W}}}\right] = 1 - \lambda \tag{1.3}$$

where Z_λ is the $100(1-\lambda)$ percentile of the standard normal distribution. Therefore if $\lambda = .025$, a 97.5% confidence interval on log yield is $(-\infty, \hat{W} + 1.96\sigma_{\hat{W}})$, or in terms of yield a 97.5% confidence interval is given by $\left(0, 10^{\hat{W}} \cdot 10^{1.96\sigma_{\hat{W}}}\right)$. Thus if $F(\sigma_{\hat{W}})$ is given by (1.1), then

$$P\left[Y < \hat{Y}\ F(\sigma_{\hat{W}})\right] = .975\ . \tag{1.4}$$

Since $\hat{W}$ will ordinarily be some averaged value, it is often referred to as the observed central value and likewise (although this is not quite correct) the corresponding $\hat{Y}$ is referred to as a central value. Thus F is said to be that multiple of the observed central value below which we are 97.5% certain the true yield falls.

There are several problems with this definition. For example, if $\sigma_{\hat{W}}$ is not known (and in fact it is not), a test for compliance cannot be made without involving distributions other than the normal. In this event the probability in Equation 1.4 is not relevant.

On the other hand, when $\tilde{W}_i$ are available, such as when CORRTEX events are available, Alevine, Gray, McCartor, and Wilson (1988) have shown that under reasonable assumptions then the test for compliance leads to a student t-distribution and in that event

$$P\left[W < \hat{W} + t_\lambda(K-1)\ S_{\hat{W}}\right] = 1 - \lambda \tag{1.5}$$

and

$$P\left[10^{W} < 10^{\hat{W}} \cdot 10^{t_\lambda(K-1)\ S_{\hat{W}}}\right] = 1 - \lambda \tag{1.6}$$

where $t_\lambda(K-1)$ is the $100(1-\lambda)$ percentile of the t-distribution with $K-1$ degrees of freedom. A more detailed discussion of this is given in Section 2 of this report.

One should note that (1.6) does not imply that (1.3) is not longer true. On the contrary, both (1.3) and (1.6) are correct when the assumptions hold. In general $t_\lambda(K-1) > Z_\lambda$ and $t_\lambda(K-1) \rightarrow Z_\lambda$ as $K \rightarrow \infty$, and therefore (1.6) does not give as tight a bound as (1.3). However, this is the penalty one pays for not knowing $\sigma_{\hat{W}}$. In terms of what we know, (taking $\lambda = .025$) all we can say is that the true yield is less than or equal to $10^{\hat{W}} \cdot 10^{t_{.025}(K-1)\ S_{\hat{W}}}$ with .975 probability. Thus it seems that in this case, the F-number should be defined as

$$F(S_{\hat{W}}) = 10^{t_{.025}(K-1)\, S_{\hat{W}}} . \tag{1.7}$$

This is exactly analogous to the motivation which led to Equation 1.1. There is a problem however, since F as defined by (1.7) is no longer a . umber but in fact is a random variable. Questions concerning whether we should consider the expected value of F, the median of F, the mode of F, etc, immediately arise.

The point is that it should be clear that the simple definition given by Equation 1.1 is inadequate. Moreover, and possibly more importantly one usually makes use of the F-number in relation to a test of compliance. In this case the real question may be, "What are our chances of detecting a violation if in fact $Y \geq Y_0$?" for some specified Y_0.

For this sort of question (1.1), (1.2) and (1.3) may not be very helpful even when $\sigma_{\hat{W}}$ is known. That is, given the F-number, the answer to the question, "What are our chances of detecting a violation when one occurs?" is certainly not obvious from (1.2) or (1.3). This is due to the fact that the question being asked is about the power of a test, whereas (1.2) and (1.3) would relate to the size of the critical region of this test. Moreover, even when (1.2) and (1.3) are correct it is doubtful that they do much more than lead to confusion since confidence intervals are commonly misunderstood.

The confusion which has arisen from defining the F-number through the confidence intervals in (1.2) and (1.3) can be seen from the testimony given by Dr. Robert Barker (the Assistant Secretary of Defense for Atomic Energy and leader of a U.S. delegation at the bilateral talks on improving verification of the TTBT) before the Senate Foreign Relations Committee on January 13, 1987. Dr. Barker was describing the types of interpretations which could be made assuming an F-number (uncertainty factor) of 2 when he testified:

" . . . this uncertainty factor means, for example, that a Soviet test for which we estimate [by the seismic method] a yield of 150 kt may have, with 95% probability [actually .95 probability], an actual

yield as high as 300 kt - twice the legal limit - or as low as 75 kt."

This statement is actually not correct. Once we have an estimated yield of 150kt Equation 1.3 says nothing about the probability. Suppose a horse has a history of winning 95% of its races. The statement is like saying the horse has a 95% chance of winning a particular race <u>after the race is over</u>. While it might be argued that we have simply been "picky" with Dr. Barker's terminology, the main problem with his *F*-number explanation is that it does not address the question which is most pertinent, i.e., "If an event is in violation, what is the chance that our techniques will detect that a violation has occurred?" In order to correct some of the problems inherent in the confidence interval definition of an *F*-number, we offer the following more general definition. It is completely compatible with the confidence interval definition.

1.2 A More General Definition of the *F*-number

As we have discussed, the definition given by Equation 1.1 is unsuitable when the standard deviation of $\hat{W}$ is unknown. Moreover the confidence interval explanation of an *F*-number is unsuitable for responding to the questions such as, "What are our chances of catching them cheating?" The following definition, first given by Alewine, et. al. (1988), alleviates these problems.

<u>Definition 1</u>: Let $\hat{W}$ be an estimate of W such that $E[\hat{W}] = W$. Suppose G is some function such that the rule: "Reject H_0 if $G(\hat{W}) \geq T_\lambda$", is a λ significance level test for the hypothesis

against

$$\left.\begin{array}{l} H_0\colon W \leq T \\ H_A\colon W > T \end{array}\right\} \tag{1.8}$$

where T is the treaty threshold and T_λ is the appropriate critical value. We then define the *F*-number of the test by

$$F_\lambda(\sigma_{\hat{W}}) = 10^{W_F - T}, \tag{1.9}$$

where W_F satisfies the equation

$$P[G(\hat{W}) > T_\lambda \mid W = W_F] = .5 \tag{1.10}$$

The probability in Equation 1.10 is called the power of the test. From Equations 1.9 and 1.10, it is seen that the F-number is the ratio of that yield for which there is a 50% chance that the hypothesis of compliance (1.8) will be rejected, to the TTBT threshold. Now note from (1.10) that

$$F_\lambda(\sigma_{\hat{W}}) \cdot 10^T = 10^{W_F} . \tag{1.11}$$

Consequently we can also state that the F-number is the multiplier of the threshold for which there is a 50% chance that the resulting true yield would be rejected as being in compliance. For example, if $T = \log 150$ kt and $F_\lambda(\sigma_{\hat{W}}) = 1.5$, then there is a 50% chance that the test would reject compliance if the true yield were 225 kt. Since G will typically be a monotonically increasing function, one can also say that if $F(\sigma_{\hat{W}}) = 1.5$, then there is more than a 50% chance of detecting a violation whenever $Y >$ 225.

Suppose now, as in Equation 1.1, we assume $\hat{W} \sim N(W, \sigma_{\hat{W}})$, where $\sigma_{\hat{W}}$ is known. Then

$$\frac{\hat{W} - W}{\sigma_{\hat{W}}} \sim N(0, 1)$$

and we can test the compliance hypothesis (henceforth we take $T = \log 150$)

$$H_0\text{: } W \leq \log 150$$

against

$$H_A\text{: } W > \log 150$$

at the λ significance level by the test: Reject H_0 if

$$\hat{W} > \log 150 + Z_\lambda \, \sigma_{\hat{W}} \, . \tag{1.12}$$

To determine the F-number for the test we follow Definition 1, i.e.

$$F_\lambda(\sigma_{\hat{W}}) = 10^{W_F - \log 150} = \frac{10^{W_F}}{150} \, ,$$

where W_F is determined from the equation

$$P[\hat{W} > \log 150 + Z_\lambda \sigma_{\hat{W}} \mid W = W_F] = .5 \, . \tag{1.13}$$

By the symmetry of the normal distribution about its mean, it follows that

$$W_F = \log 150 + Z_\lambda \sigma_{\hat{W}}$$

and hence (1.14)

$$F_\lambda(\sigma_{\hat{W}}) = 10^{Z_\lambda \sigma_{\hat{W}}} \, .$$

Note however that obtaining $F_\lambda(\sigma_{\hat{W}})$ from (1.13) has an immediate advantage. That is, given a value for $F_\lambda(\sigma_{\hat{W}})$ in (1.14), and a desired significance level, $\sigma_{\hat{W}}$ is determined by Equation 1.14 and the probability of detecting a violation for any given W, say $W = W_1$, is given by the left side of Equation (1.13) when W_F is replaced by W_1. Thus if $\lambda = .025$ and $F_\lambda(\sigma_{\hat{W}}) = 2$, then it follows that $\sigma_{\hat{W}} = .159$. If one desires to know the probability that we would detect a violation under these conditions when say, $W = 400$kt, one simply substitutes in the left side of Equation 1.13 to obtain

$$P[\hat{W} > \log 150 + 1.96\sigma_{\hat{W}} \mid W = \log 400 \,] = .79$$

The probability defined by the left side of Equation 1.13 is called the power of the test. In words, it is the probability that the hypothesis H_0 will be rejected for a specified value of the true log yield W. A short table of values of the power of the test defined by Equation 1.12 for various values of W and F

number is given in Table 1.

TABLE 1 - Power for Various F-Numbers and True Yields

$\lambda = .025$

F-Number

True Yield	1.3	1.5	1.8	2.0
175	0.209	0.112	0.074	0.064
195	0.500	0.245	0.139	0.112
225	0.857	0.500	0.272	0.208
270	0.992	0.811	0.500	0.383
300	0.999	0.918	0.637	0.500
350	1.000	0.984	0.807	0.669
400	1.000	0.997	0.905	0.792
450	1.000	1.000	0.956	0.874

1.3 The Unknown Variance Case

Suppose now that $\sigma_{\hat{W}}$ is unknown but that $S_{\hat{W}}$, independent of $\hat{W}$, is available as in Equation 1.5. We previously remarked concerning the ambiguity that arises from (1.7). We will now show that Definition 1 removes that ambiguity. In order to do so we first define a test for compliance which is more fully discussed in the latter sections of this report. In this case $(\hat{W} - W)/S_{\hat{W}}$ is distributed as a Student's t-distribution with $K-1$ degrees of freedom, i.e. as $t(K-1)$. Thus we have the following test at the .025 significance level. Reject H_0: $W_i \leq \log 150$ if $\hat{W}_i > \log 150 + t_{.025}(K-1)\ S_{\hat{W}}$. Following Definition 1 we can now find the F number for the test. We need to determine W_F such that

$$P\left[\hat{W} > \log 150 + t_{.025}(K-1)\ S_{\hat{W}} \mid W = W_F\right] = .5$$

or equivalently

$$P\left[\frac{\hat{W} - \log 150}{S_{\hat{W}}} > t_{.025}(K-1) \mid W = W_F\right] = .5\ . \qquad (1.15)$$

However, when $W = W_F \neq \log 150$, $(\hat{W} - \log 150)/S_{\hat{W}}$ is distributed as a noncentral t and a closed form for W_F cannot be given. However to a very good approximate solution (to several decimal places), to Equation 1.15 is given by

$$t_{.025}(K-1) = \frac{W_F - \log 150}{E(S_{\hat{W}})} \tag{1.16}$$

(Alewine, et. al, 1988). From (1.16) it follows that

$$W_F = \log 150 + t_{.025}(K-1)\ E(S_{\hat{W}}). \tag{1.17}$$

Thus, we have approximately

$$F_{.025}(\sigma_{\hat{W}}) = 10^{t_{.025}(K-1)\ E(S_{\hat{W}})}. \tag{1.18}$$

Note that the F-number in (1.18) is not a random variable as in Equation 1.7, but is approximately the expected value of the F-number in Equation 1.7. Since in general $E[S_{\hat{W}}] = c\sigma_{\hat{W}}$ for some c, the F-number in (1.18) can be written as

$$F_{.025}(\sigma_{\hat{W}}) = 10^{t_{.025}(K-1)\ c\sigma_{\hat{W}}}. \tag{1.19}$$

This should be compared to Equation 1.1. Since in general $ct_{.025}(K-1) > 1.96$, the F-number as given by Equation 1.19 is larger than the F-number given by Equation 1.1.

The case where $\sigma_{\hat{W}}$ is not known but can be estimated if the ratio of the CORRTEX to the seismic variance is known or small was studied by Alwine, et. al. (1988). In that case, $S_{\hat{W}}$ defined here, is given by

$$S_{\hat{W}} = \tau\ S_u\ /\ B,$$

where B, τ and S_u are as defined in by Alewine, et. al. (1988).

In summary, several points should now be clear.

1. The definition of the F-number given by Equation 1.1 should only be used when the assumption of known variance is justified, and this definition of the F-number is physically meaningful only in this setting.
2. The confidence interval motivation for the F-number defined by Equation 1.1 is useful when addressing significance level questions, i.e. false alarm rate questions but will usually lead to confusion regarding power questions, i.e. questions concerning our chances of detecting noncomplying events.
3. The F-number defined by Definition 1 is equivalent to the F-number defined by

Equation 1.1 when $\sigma_{\hat{W}}$ is known. Moreover, the recommended presentation of the F-number makes it more suitable for addressing power questions, i.e. questions concerning the Soviets' cheating.

4. The F-number defined by Definition 1 is entirely general regarding the single event question. That is, the F-number defined by Definition 1 is appropriate whether or not $\sigma_{\hat{W}}$ is known and whether or not the normality assumption is valid.

1.4 F-number for Biased Estimates

In Definition 1, we assumed that $E[\hat{W}] = W$ and as a result we considered the F-number as a measure of the precision of the test depending only on $\sigma_{\hat{W}}$ and λ. This was reflected in our notation $F_\lambda(\sigma_{\hat{W}})$. However it may be that $E[\hat{W}] = W - b$, $b > 0$, i.e. it may be that our estimator underestimates W on a systematic basis. In this event the power and the significance level of the test would be reduced. The result of the power being reduced is that the F-number as given by Definition 1 would be too small. However it is an easy matter to correct this problem. This is the purpose of the following definition which is an extension of Definition 1 that does not require $E[\hat{W}] = W$.

Definition 2

Let $\hat{W}$ be an estimate of W such that $E[\hat{W}] = W - b$, $b \geq 0$. Suppose G is a function of $\hat{W}$ and T_λ is a given value such that the rule: Reject H_0 if $G(\hat{W}) > T_\lambda$, is a λ level significance test for the hypothesis

$$\begin{aligned} &H_0: W \leq T \\ \text{against} \quad &H_A: W > T \end{aligned} \tag{1.20}$$

when $b = 0$, and is an $\lambda_1 \leq \lambda$ significance level test when $b \geq 0$. We then define the F-number of the test by

$$F_\lambda(\sigma_{\hat{W}}, b) = 10^{W_F - T} \ , \tag{1.21}$$

where W_F satisfies the equation

$$P[G(\hat{W}) > T_\lambda \mid W = W_F] = .5 \ . \tag{1.22}$$

This is of course the same as Definition 1, with the exception that we no longer require $b = 0$.

Consider once again the known variance case for testing the hypothesis in Equation 1.8. In this event the test is: Reject H_0 if $\hat{W} > \log 150 + Z_\lambda \sigma_{\hat{W}}$. Now suppose $E[\hat{W}] = W - b$, $b > 0$. Then the test of H_0 will have a true significance level of $\lambda_1 < \lambda$ and the F-number will be larger than $F_\lambda(\sigma_{\hat{W}})$. We can apply Definition 2 to determine the effect of the bias, b, on the F-number. By Definition 2 we want to find an W_F such that

$$P[\hat{W} > \log 150 + Z_\lambda \sigma_{\hat{W}} \mid W = W_F] = .5 .$$

Then

$$P[\hat{W} - (W_F - b) > \log 150 - W_F + b + Z_\lambda \sigma_{\hat{W}} \mid W = W_0] = .5 . \tag{1.23}$$

However, from (1.23) and the symmetry of $\hat{W}$ about its mean, $W-b$, it follows that

$$\log 150 - W_F + b + Z_\lambda \sigma_{\hat{W}} = 0 .$$

Therefore

$$W_F = \log 150 + b + Z_\lambda \sigma_{\hat{W}}$$

and

$$F_\lambda(\sigma_{\hat{W}}, b) = 10^{b + Z_\lambda \sigma_{\hat{W}}}$$

$$= 10^b \, 10^{Z_\lambda \sigma_{\hat{W}}}$$

$$= F(b) \, F_\lambda(\sigma_{\hat{W}}) , \tag{1.24}$$

where

$$F(b) = 10^b . \tag{1.25}$$

Note that now the precision of the test is effected by two factors, $F(b)$ and $F_\lambda(\sigma_{\hat{W}})$. We refer to $F(b)$ as the *F due to statistical bias* and $F_\lambda(\sigma_{\hat{W}})$ as *F due to variance*. Unless it is clear that $\hat{W}$ is a biased estimator, we simply refer to the F due to variance as the F-number.

1.5 The F-number for testing Compliance for a Set of Events

In everything we have considered so far, we have defined the F-number for determining compliance of a simple event. These ideas are not directly extendable to testing compliance of a set of events. This is not a shortcoming of our definition but simply the consequence of the fact that for a set of events there is no unique way for the set to be out of compliance. In order to obtain a unique F-number for a set of events it would therefore be necessary to define a probability distribution on the possible values of W, i.e. a Bayesian approach is required. Since there seems to be no basis for determining such a distribution we will not pursue this question at this time.

Section 2

Testing Compliance of an Event when CORRTEX is not Available, Based on Data From k Events for which Both Seismic and CORRTEX are Available

2.1 Introduction

In this section, we consider tests for compliance introduced by Alewine et. al. (1988). In that report, it was suggested that if past CORRTEX events were available it might be better to base compliance tests on the assumption that the ratio of the CORRTEX variance to the seismic variance is known rather than to base the test on the assumption that the individual variances are known. This conjecture is investigated here and from a robustness point of few it is demonstrated that the assumption of the ratio is indeed preferable. The need for the more general definition of the F-number proposed in the previous section will be clear in this section.

The basic setting which will be discussed here is the situation in which there are k events for which both magnitude and CORRTEX readings are available. Based on these data, tests are then

developed for testing the hypothesis that a new event, for which only seismic information is available, is in compliance. That is, we test the null hypothesis that the yield for the new event is less than or equal to 150 kt. Throughout this report the following notation will be used:

m_i = the magnitude measurement for event i

Y_i = the yield for event i

$W_i = \log\ Y_i$

A = true geographic bias

$\hat{A}$ = estimated geographic bias

B = slope

$\hat{W}_i$ = estimated log yield for the ith event based on seismic readings of magnitude

$\tilde{W}_i$ = estimated log yield for the ith event based on CORRTEX readings

It will be assumed that log yield and magnitude of the ith event are related by

$$m_i = A + B\ W_i + \epsilon_i\ , \qquad \epsilon_i \sim N\left(0,\ \sigma^2_{\mathrm{SEI}}\right), \tag{2.1}$$

where B is known. If CORRTEX data is available on event i, then it is also assumed that

$$\tilde{W}_i = W_i + e_i\ , \qquad e_i \sim N\left(0,\ \sigma^2_{\mathrm{COR}}\right). \tag{2.2}$$

In the current setting we assume that m_i and $\tilde{W}_i$ are available for events $i = 1,\ \ldots\ ,\ k$. We further assume that ϵ_i and e_i are independent. Then based on these k readings, an unbiased estimator of A is given by

$$\hat{A} = \frac{1}{k}\sum_{i=1}^{k}\left(m_i - B\tilde{W}_i\right). \tag{2.3}$$

We now consider a new event, denoted with the subscript k+1, for which only magnitude information

is available. Based upon the new magnitude reading, m_{k+1}, an unbiased estimate of log yield is given by

$$\hat{W}_{k+1} = \frac{m_{k+1} - \hat{A}}{B} . \tag{2.4}$$

Denoting the variance of $\hat{W}_{k+1}$ by $\sigma^2_{\hat{W}}$, we have

$$\sigma^2_{\hat{W}} = \left(1 + \frac{1}{k}\right) \frac{\sigma^2_{\mathrm{SEI}}}{B^2} + \frac{\sigma^2_{\mathrm{COR}}}{k} . \tag{2.5}$$

Two cases will be considered:

I. σ_{SEI} and σ_{COR} are known

II. $\sigma_{\mathrm{SEI}}/\sigma_{\mathrm{COR}}$ is known

We will also consider the performance of the tests in Case I and Case II when only approximations to σ_{SEI} and COR are available.

2.2 Compliance Tests

Case I. Suppose that σ_{SEI} is known. Then $\hat{W}_{k+1}$ defined in (2.4) is normal with variance $\sigma^2_{\hat{W}}$ given by (2.5) and the .025 level complicance test is: Reject compliance if:

$$\hat{W}_{k+1} > \log 150 + 1.96 \left[\left(1 + \frac{1}{k}\right) \frac{\sigma^2_{\mathrm{SEI}}}{B^2} + \frac{\sigma^2_{\mathrm{COR}}}{k}\right]^{1/2} . \tag{2.6}$$

The test for any given significance level, λ, is:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + Z_{\lambda} \left[\left(1 + \frac{1}{k}\right) \frac{\sigma^2_{\mathrm{SEI}}}{B^2} + \frac{\sigma^2_{\mathrm{COR}}}{k}\right]^{1/2} , \tag{2.7}$$

where Z_λ is the 100(1-λ) percentile of a N(0,1) distribution. From (2.7) the F-number for Case I is given by

$$F_\lambda(\sigma_{\hat{W}}) = 10^{Z_\lambda\left[\left(1+\frac{1}{k}\right)\frac{\sigma^2_{\mathrm{SEI}}}{B^2}+\frac{\sigma^2_{\mathrm{COR}}}{k}\right]^{1/2}} \tag{2.8}$$

In Table 2, we display the F-number for Case I for $\sigma_{\mathrm{COR}} = .04$, and various values of σ_{SEI}.

<u>Case II.</u> To consider Case II, let

$$R^2 = \left[\frac{\sigma^2_{\mathrm{COR}}}{\frac{\sigma^2_{\mathrm{SEI}}}{B^2}+\sigma^2_{\mathrm{COR}}}\right]$$

and

$$\tau^2 = 1 + \frac{1}{k} - R^2 \tag{2.8}$$

$$= \frac{\sigma^2_{\mathrm{SEI}}/B^2}{\sigma^2_{\mathrm{SEI}}/B^2+\sigma^2_{\mathrm{COR}}} + \frac{1}{k}\,. \tag{2.9}$$

Then, under the hypothesis of compliance, if $r = \sigma^2_{\mathrm{COR}}/\sigma^2_{\mathrm{SEI}}$ is known, then R is known and

$$\frac{\hat{W}_{k+1} - \log 150}{\frac{\tau\, S_u}{B}} \sim t(k-1)\,, \tag{2.10}$$

where

$$S_u^2 = \frac{1}{k-1}\sum_{i=1}^{k}\left(u_i - \bar{u}\right)^2, \tag{2.11}$$

with $u_i = m_i - B\tilde{W}_i$ and $t(k-1)$ a Student's t random variable with $k-1$ degrees of freedom. The resulting test for compliance is given by:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + t_\lambda(k-1)\ \tau\ S_u/B\ , \tag{2.12}$$

where $t_\lambda(k-1)$ is the $100(1-\lambda)$ percentile of the t-distribution with $k-1$ degrees of freedom. A short listing of t values for λ=.05 and λ=.025 follows:

k	$t_{.05}(k-1)$	$t_{.025}(k-1)$
2	6.314	12.706
3	2.920	4.303
4	2.353	3.183
5	2.132	2.776
10	1.833	2.262

A more extensive table for the t distribution can be found in almost any introductory book in statistics. From Definition 1 of Section 1, the F-number for Case II, is given by

$$F_\lambda(\sigma_{\dot{W}}) = 10^{W_F - T}\ , \tag{2.13}$$

where W_F satisfies the equation

$$P\left[\hat{W}_{k+1} > \log 150 + t_\lambda(k-1)\,\frac{\tau\ S_u}{B} \mid W = W_F\right] = .5 \tag{2.14}$$

For k = 2 or 3, this equation can be solved for W_F numerically. However for $k > 3$, a very simple approximate solution to (2.14) is given by

$$W_F = \log 150 + t_\lambda(k-1)\ \tau\ E[S_u]\ /\ B\ . \tag{2.15}$$

Therefore, for $T = \log 150$,

$$F_\lambda(\sigma_{\dot W}) = 10^{t_\lambda(k-1)\ \tau\ E[S_u]\ /\ B} . \tag{2.16}$$

Now

$$E[S_u] = \left(\frac{2}{k-1}\right)^{1/2} \frac{\Gamma\left(\frac{k}{2}\right)}{\Gamma\left(\frac{k-1}{2}\right)}\ \tau\ \sigma_{\dot W}$$

and

$$\left(\frac{2}{k-1}\right)^{1/2} \frac{\Gamma\left(\frac{k}{2}\right)}{\Gamma\left(\frac{k-1}{2}\right)} \approx \frac{4(k-1)}{4k-2.75} ,$$

so that to a very good approximation,

$$F_\lambda(\sigma_{\dot W}) = 10^{t_\lambda(k-1)\frac{4(k-1)}{4k-2.75}\ \tau\ \left(\sigma^2_{\mathrm{SEI}} + B^2\sigma^2_{\mathrm{COR}}\right)^{1/2}/\mathrm{B}} . \tag{2.17}$$

Actually, the approximation in (2.17) is good to approximately two decimal places for $k > 2$. (see Alewine, et. al. (1988).

In Table 3 we show the F-numbers found using (2.17) for the parameter configurations considered in Table 2. There it can be seen that the F-numbers for Case II tend to be slightly larger than those for Case I.

Note that in Case II the necessity for the more general definition of an F-number is clear. As we noted in Section 1, had we used the confidence interval definition of the F-number we would have obtained

$$F^{(\mathrm{CI})}_\lambda(\sigma_{\dot W}) = 10^{t_\lambda(k-1)\ \tau\ S_u/\ B} \tag{2.18}$$

which is in fact not a number at all but is a random variable!

2.3 Robustness of the Compliance Test

In the compliance test outlined in the previous pages, it was necessary to assume either that both σ^2_{COR} and σ^2_{SEI} are known or that their ratio is known. In reality such parameters will not be known

exactly but instead we will have to use our best estimates or best guess of them. This of course introduces some imprecision into our probability statements. Consideration of the impact of such assumption errors are referred to as robustness studies. In this subsection, we will consider the implications of $\sigma_{SEI} \neq \tilde{\sigma}_{SEI}$, where we now use $\sim$ to distinguish between the true value of the parameter and the assumed value, $\sim$ denoting the assumed value. We will continue to assume that σ_{COR} is known, although results for σ_{COR} unknown could also be obtained.

In this setting the test corresponding to (2.7) for Case I is:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + Z_{\lambda}\left[(1+\tfrac{1}{k})\frac{\tilde{\sigma}^2_{SEI}}{B^2} + \frac{\sigma^2_{COR}}{k}\right]^{1/2} \qquad (2.19)$$

while the test based on the ratio is given by:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + t_{\lambda}(k-1)\,\tilde{r}\,S_u/B\,. \qquad (2.20)$$

In Tables 4 and 5, we display the actual significance levels of the Case I and Case II tests, respectively for various combinations of σ^2_{SEI} and $\tilde{\sigma}_{SEI}$ in the case $\lambda = .025$ and $\sigma_{COR} = .04$. There it can be seen that if $\tilde{\sigma}_{SEI} \geq \sigma_{SEI}$, then both tests are $\lambda_1 \leq \lambda$ level tests. Conversely, when $\tilde{\sigma}_{SEI} \leq \sigma_{SEI}$, both tests are $\lambda_1 \geq \lambda$ level tests. Note that the effect on true significance level is not nearly as dramatic for Case II (assuming only the ratio to be known) than for Case I (assuming both σ_{SEI} and σ_{COR} to be known). An explanation for the fact that the Case II test is not as sensitive to misspecification of the variances is that $E(S_u^2) = \sigma^2_{SEI} + B^2\sigma^2_{COR}$, and thus S_u provides information from the data concerning the true value of σ_{SEI} when σ_{COR} is known. .

Expressions for the F-numbers for the two tests based on imperfect knowledge of σ_{SEI} for Case I by

$$F_{\lambda}(\sigma_{\hat{W}}) = 10^{Z_{\lambda}\left[(1+\frac{1}{k})\frac{\tilde{\sigma}^2_{\mathrm{SEI}}}{B^2} + \frac{\sigma^2_{\mathrm{COR}}}{k}\right]^{1/2}} \tag{2.21}$$

and for Case II by

$$F_{\lambda}(\sigma_{\hat{W}}) = 10^{t_{\lambda}(k\text{-}1)\ \tilde{\tau}\ \mathrm{E}(S_u)/B}$$

which can be approximated as in (2.17) for $k > 2$ by

$$F_{\lambda}(\sigma_{\hat{W}}) = 10^{t_{\lambda}(k-1)\frac{4(k-1)}{4k-2.75}\frac{\tilde{\tau}}{B}(\sigma^2_{\mathrm{SEI}} + B^2\sigma^2_{\mathrm{COR}})^{1/2}} \,. \tag{2.22}$$

It is interesting to note that the F-number for Case I depends only on the estimated value for σ_{SEI} and does not depend on the true value. For this reason the F-numbers in the present setting can be found from Table 2 by taking σ_{SEI} in the table to be the estimated value. The F-number in (2.22) for Case II with imperfect knowledge depends on both the estimated value (through $\tilde{\tau}$) and the true-value (through $\sigma^2_{\mathrm{SEI}} + B^2\sigma^2_{\mathrm{COR}}$).

In Table 6, we show the F-numbers for the Case II test for the same parameter configurations considered in Tables 4 and 5. Several observations should be made from the tables:

(1) For both tests we see that there is a trade-off between true significance level, and F-number. Specifically, whenever $\tilde{\sigma}_{\mathrm{SEI}} \geq \sigma_{\mathrm{SEI}}$, the true significance level, λ_1, is less than or equal to λ but at a cost of a larger F-number. On the other hand, whenever $\tilde{\sigma}_{\mathrm{SEI}} < \sigma_{\mathrm{SEI}}$, the F-numbers are reduced but $\lambda_1 > \lambda$, i.e. the test no longer has the desired false alarm rate.

(2) If σ_{SEI} and σ_{COR} are truly known, then Case I gives a substantially smaller F-number than Case II for a small number of CORRTEX events, k. However for k as large as 5 or 6 the F-numbers for the two cases are not substantially different.

(3) The significance level of Case I is dramatically effected by errors in approximating σ_{SEI}. For example, if $\sigma_{\mathrm{SEI}} = .08$ and $\tilde{\sigma}_{\mathrm{SEI}} = .05$, then the true significance level is approximately .1 for $2 \leq k \leq 7$ and increases to about .11 at $k = 20$. Since the advertised level is .025, this is a substantial error. On the otherhand in Case II, if $\tilde{\sigma}_{\mathrm{SEI}} = .05$ when $\sigma_{\mathrm{SEI}} = .08$, the significance level for $2 \leq k \leq 7$ is around .035 and slowly increases to .04 at $k = 20$. On the otherhand if $\tilde{\sigma}_{\mathrm{SEI}} = .08$ and $\sigma_{\mathrm{SEI}} = .05$, in Case I the significance level is .001 for essentially all k, whereas in Case II, the significance level is around .02 for reasonable values of k. It is therefore very clear, that if CORRTEX is available, Case II offers a substantially more robust test.

(4) The F-numbers for Case II tend to be lower than those for Case I when $\tilde{\sigma}_{\mathrm{SEI}} > \sigma_{\mathrm{SEI}}$. This corresponds to the fact that in these cases the significance levels for the Case I test tend to be substantially smaller than the nomial $\lambda = .025$ level. On the otherhand when $\tilde{\sigma}_{\mathrm{SEI}} < \sigma_{\mathrm{SEI}}$, the F-numbers for the Case I test tend to be smaller than those for Case II. However, in these cases it should be recalled that the observed significance levels for the Case I tests were often very high. The fact that the F-number is small is irrelevant if the false alarm rate is unacceptably high.

2.4 A Modified Case II Test

For the Case I and Case II tests, the conservative approach is to specify $\tilde{\sigma}_{\mathrm{SEI}}$ in such a way that $\tilde{\sigma}_{\mathrm{SEI}} \geq \sigma_{\mathrm{SEI}}$. Whenever $k > 2$ CORRTEX events are available, a test can be obtained which always has true significance level less than or equal to λ and which does not require σ_{SEI}, σ_{COR} nor their ratio to be specified. In this case we take $\tilde{\tau}$ to be $\left(1 + \frac{1}{k}\right)^{1/2}$, and we will denote this by τ_0 to emphasize the fact that it is the value of τ in (2.8) associated with $R = 0$. The test becomes:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + t_{\lambda}(k-1) \left(1 + \frac{1}{k}\right)^{1/2} S_u/B \,. \qquad (2.23)$$

It is easy to see that $\tilde{\tau} < \tilde{\tau}_0$ for all positive values of $\tilde{\sigma}_{\mathsf{SEI}}$ and σ_{COR} and that the test in (2.23) approximates the test in (2.12) when $\sigma_{\mathsf{SEI}} \gg \sigma_{\mathsf{COR}}$. The test can be thought of as a Case II test with $\tilde{\sigma}_{\mathsf{SEI}} = \infty$ or $\sigma^2_{\mathsf{COR}} = 0$. It follows immediately that the test in (2.23) is an $\lambda_1 \leq \lambda$ significance level test. One can show that to a very good approximation, the F-number corresponding to (2.23) is given for $k > 2$ by

$$F_\lambda(\sigma_{\dot{W}}) = 10^{t_\lambda(k-1)\frac{4(k-1)}{4k-2.75}(1+k)^{1/2}\left(\sigma^2_{\mathsf{SEI}} + B^2\sigma^2_{\mathsf{COR}}\right)^{1/2}/B} \quad . \tag{2.24}$$

Since $\tilde{\tau}_0 \geq \tilde{\tau}$ it follows that the F-number for the Case II test (either with variances known or unknown) is always less than or equal to the corresponding F-number for the modified Case II test. In Tables 7 and 8 we show F-numbers and significance levels, respectively, for this modified Case II test. There it can be seen that the significance levels are always less than or equal to the nominal level of $\lambda = .025$ while the F-numbers tend to be larger than those shown for Case I and Case II tests. Thus, the modified Case II test provides a conservative alternative in the cases in which a good a priori bound on σ_{SEI} is not available.

Section 3

Estimating σ_{SEI} from Seismic Data

3.1 Estimation Based on Events for which both Seismic and CORRTEX Data are Available

The tests discussed in Sections 1 and 2 were based on an a priori value for σ_{SEI}. However, it is possible to obtain an estimate of σ_{SEI} based on the k (>1) shots for which both seismic and CORRTEX readings are available if σ_{COR} is assumed to be known. Under this assumption, since $E[S^2_u] = \sigma^2_{\mathsf{SEI}} + B^2\sigma^2_{\mathsf{COR}}$, where S^2_u is given in (2.11), it follows that σ^2_{SEI} can be estimated as

$$\hat{\sigma}^2_{\mathsf{SEI}} = [S^2_u - B^2\sigma^2_{\mathsf{COR}}] \text{ , if } S^2_u \geq B^2\sigma^2_{\mathsf{COR}} \tag{3.1}$$

$$= 0 \ , \ \text{if } S_u^2 < B^2\sigma^2_{\mathrm{COR}} \ .$$

Of course, the estimator in (2.24) will be poor when k is small. However, this estimate does utilize information from the k observations concerning the value of σ^2_{SEI}. The obvious modification of (2.20) is to substitute $\hat{\sigma}_{\mathrm{SEI}}$ for $\tilde{\sigma}_{\mathrm{SEI}}$ in $\tilde{\tau}$ to obtain:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + t_\lambda(k-1) \left(\frac{\hat{\sigma}^2_{\mathrm{SEI}}/B^2}{\hat{\sigma}^2_{\mathrm{SEI}}/B^2 + \sigma^2_{\mathrm{COR}}} + \frac{1}{k} \right)^{1/2} S_u/B \ . \tag{3.2}$$

Although we have been unable to calculate theoretical significance levels and F-numbers for the test in (3.2), simulations were run for the case $\sigma_{\mathrm{COR}} = .04$ and $B = 1$ in order to estimate the F-numbers and true significance levels associated with this test. The empirical estimates of F can be derived from empirical power. The Case I test can also be modified in this situation to give the test:

Reject compliance if

$$\hat{W}_{k+1} > \log 150 + Z_\lambda \hat{\sigma}_{\hat{W}} \tag{3.3}$$

where $\hat{\sigma}^2_{\hat{W}} = (1 + \frac{1}{k})\hat{\sigma}^2_{\mathrm{SEI}} + \frac{1}{k}\sigma^2_{\mathrm{COR}}$. Preliminary results indicate that for larger values of k, the tests in (3.2) and (3.3) have significance levels somewhat above nominal levels over the entire range of possible σ_{SEI} values. Additionally the F-numbers appear to be competitive with those obtained by the other tests. These results also show that the modified Case I test in (3.3) has somewhat higher significance levels than those obtained for the test in (3.2). However, the significance levels for (3.3) did not reach the excessively high levels observed in Table 4 for the Case I test. The estimate of σ^2_{SEI} from (3.1) assumes that σ_{COR} is known. If this is in fact not the case, then simulations similar to

those mentioned here can be run to determine the effect of imperfect knowledge of σ_{COR}. In this case we expect the modified Case II test to be more robust. Another possible modification of the tests would be based on the use of a weighted estimate of σ^2_{SEI}, which uses both a priori and estimated information concerning σ^2_{SEI}. The simple proposed estimator is given by

$$\hat{\hat{\sigma}}^2_{SEI} = \frac{a_1\tilde{\sigma}^2_{SEI} + a_2\hat{\sigma}^2_{SEI}}{a_1 + a_2}$$

where a_1 and a_2 are constants picked based on physical considerations. We believe that further investigation is warranted into the modification of the Case I and Case II compliance tests to make use of the seismic and CORRTEX data for estimation of σ^2_{SEI}.

3.2 Estimation Based on a Mixture Model for Seismic Data

Although the estimator in (3.1) provides a method for estimating σ_{SEI} from data, to date there have only been $k = 1$ event for which both seismic and CORRTEX data are available. Only when more data of this type become available will the use of (3.1) be worthwhile. Gray, Woodward and McCartor (1989) developed techniques which provide an estimate of σ_{SEI} from seismic data alone by modeling magnitude (or equivalently log-yield) as a mixture of normal components. A random variable, X, is said to be distributed as a mixture of normals if its probability density function f is given by

$$f(x; p, \mu, \sigma) = \sum_{k=1}^{l} \frac{p_k}{\sqrt{2\pi}\ \sigma_k} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_k}{\sigma_k}\right)^2\right], \tag{3.4}$$

where $\sum_{k=1}^{l} p_k = 1$, $p_k \geq 0$. In our application, the assumption of a common component standard deviation, i.e. $\sigma_i \equiv \sigma$, is a reasonable one. The maximum likelihood estimates are given as the iterative solution of the following equations:

$$\hat{p}_k^{(m)} = \frac{\hat{p}_k^{(m-1)}}{n} \sum_{i=1}^{n} \frac{f_k^{(m-1)}(x_i)}{f^{(m-1)}(x_i)} \tag{3.5}$$

$$\hat{\mu}_k^{(m)} = \frac{\frac{1}{n} \sum_{i=1}^{n} \left\{ x_i \frac{f_k^{(m-1)}(x_i)}{f^{(m-1)}(x_i)} \right\}}{\sum_{i=1}^{n} \left\{ \frac{f_k^{(m-1)}(x_i)}{f^{(m-1)}(x_i)} \right\}}, \quad k = 1, 2, \ldots, l, \tag{3.6}$$

$$\hat{\sigma}^{2(m)} = \frac{1}{n} \sum_{i=1}^{n} \sum_{k=1}^{l} \left\{ \hat{p}_k^{(m-1)} (x_i - \hat{\mu}_k^{(m-1)})^2 \frac{f_k^{(m-1)}(x_i)}{f^{(m-1)}(x_i)} \right\}. \tag{3.7}$$

where m denotes the mth iterate while $f^{(m)}$ and $f_k^{(m)}$ represent the mth iterate of the mixture density given in (3.4) and the kth component density

$$f_k(x) = \frac{1}{\sqrt{2\pi}\,\hat{\sigma}} \exp\left[-\frac{1}{2} \left(\frac{x - \hat{\mu}_k}{\hat{\sigma}} \right)^2 \right] \tag{3.8}$$

respectively.

It is not unreasonable to expect that more than one explosion would be made at (roughly) each of several theoretical yield levels associated with the weapons being developed. Also, since the levels of testing associated with different weapons are likely to differ significantly, one may expect the components to be sufficiently well separated. Thus, if the mixture random variable X in (3.4) is magnitude, then $\sigma = \sigma_{\mathrm{SEI}}$.

In order to determine how well the component standard deviation can be estimated, we simulated samples from mixtures of normals whose common component variances, σ^2, are known and for which the mixing proportions are approximately equal. The component means take on the values $2.176 - (k-1)d\sigma$, $k = 1, 2, \ldots, l$, where d is a multiplier specifying the separataion among the

components and $\sigma = .06$. Note that $\mu_{\max} = 2.176$ in all cases considered in this section so that these are situations in which the null hypothesis of compliance is true. We consider the cases in which the number of components, l, is 2, 3, and 4 and in which the multiplier d takes on the values 1.5, 2, 2.5 and 5. For each of the 12 resulting combinations we independently generated 200 samples of size $n = 80$. In Table 9 we show the bias and $\sqrt{\text{MSE}}$ associated with the estimation of σ_{SEI}, given by

$$\text{bias} = \sum_{i=1}^{200} \frac{\hat{\sigma}_{\text{SEI}}^{(i)}}{200} - .06$$

and

$$\text{MSE} = \sum_{i=1}^{200} \frac{(\hat{\sigma}_{\text{SEI}}^{(i)} - .06)^2}{200}$$

where $\hat{\sigma}_{\text{SEI}}^{(i)}$ denotes the estimate of σ_{SEI} for the ith sample. There it can be seen that, as would be expected, the quality of the estimates of σ_{SEI} improve as separation among components increases. However, for separations of 2.5σ or less, there was substantial variability in the estimate of σ_{SEI}. The results of Table 9 indicate that estimates from the mixture-of-normals approach can provide rough bounds on σ_{SEI}.

REFERENCES

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Table 2

F-Numbers for Case I

Lambda = 0.025 B = 1
Sigma Cor = 0.040

Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	1.23	1.29	1.36	1.43	1.50	1.58	1.67
3	1.21	1.26	1.32	1.39	1.46	1.54	1.62
4	1.19	1.25	1.31	1.37	1.44	1.51	1.59
5	1.18	1.24	1.30	1.36	1.43	1.50	1.57
6	1.18	1.23	1.29	1.35	1.42	1.49	1.56
7	1.17	1.23	1.28	1.35	1.41	1.48	1.55
8	1.17	1.22	1.28	1.34	1.41	1.47	1.55
9	1.17	1.22	1.28	1.34	1.40	1.47	1.54
10	1.17	1.22	1.28	1.34	1.40	1.47	1.54
11	1.16	1.22	1.27	1.33	1.40	1.46	1.53
12	1.16	1.22	1.27	1.33	1.40	1.46	1.53
13	1.16	1.21	1.27	1.33	1.39	1.46	1.53
14	1.16	1.21	1.27	1.33	1.39	1.46	1.53
15	1.16	1.21	1.27	1.33	1.39	1.46	1.53
16	1.16	1.21	1.27	1.33	1.39	1.45	1.52
17	1.16	1.21	1.2	1.33	1.39	1.45	1.52
18	1.16	1.21	1.27	1.33	1.39	1.45	1.52
19	1.16	1.21	1.27	1.32	1.39	1.45	1.52
20	1.16	1.21	1.26	1.32	1.39	1.45	1.52

Table 3

F-Numbers for Case II

Lambda = 0.025 B = 1
Sigma Cor= 0.040

Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.43	1.56	1.70	1.87	2.06	2.26	2.49
4	1.30	1.38	1.48	1.59	1.71	1.84	1.98
5	1.25	1.32	1.41	1.50	1.59	1.70	1.81
6	1.22	1.29	1.37	1.45	1.54	1.63	1.73
7	1.21	1.27	1.35	1.42	1.51	1.59	1.68
8	1.20	1.26	1.33	1.40	1.48	1.57	1.65
9	1.19	1.25	1.32	1.39	1.47	1.55	1.63
10	1.19	1.25	1.31	1.38	1.45	1.53	1.62
11	1.18	1.24	1.31	1.37	1.45	1.52	1.60
12	1.18	1.24	1.30	1.37	1.44	1.51	1.59
13	1.18	1.23	1.30	1.36	1.43	1.51	1.58
14	1.17	1.23	1.29	1.36	1.43	1.50	1.58
15	1.17	1.23	1.29	1.35	1.42	1.50	1.57
16	1.17	1.23	1.29	1.35	1.42	1.49	1.57
17	1.17	1.22	1.28	1.35	1.42	1.49	1.56
18	1.17	1.22	1.28	1.35	1.41	1.48	1.56
19	1.17	1.22	1.28	1.34	1.41	1.48	1.55
20	1.16	1.22	1.28	1.34	1.41	1.48	1.55

Table 4

Actual Significance Levels for Case I
when True Variances are Unknown

Lambda = 0.025 B = 1
Sigma Cor = 0.040

True Sigma Sei = 0.030

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.025	0.008	0.002	0.000	0.000	0.000	0.000
3	0.025	0.008	0.002	0.000	0.000	0.000	0.000
4	0.025	0.007	0.001	0.000	0.000	0.000	0.000
5	0.025	0.007	0.001	0.000	0.000	0.000	0.000
6	0.025	0.006	0.001	0.000	0.000	0.000	0.000
7	0.025	0.006	0.001	0.000	0.000	0.000	0.000
8	0.025	0.006	0.001	0.000	0.000	0.000	0.000
9	0.025	0.006	0.001	0.000	0.000	0.000	0.000
10	0.025	0.006	0.001	0.000	0.000	0.000	0.000
11	0.025	0.006	0.001	0.000	0.000	0.000	0.000
12	0.025	0.005	0.001	0.000	0.000	0.000	0.000
13	0.025	0.005	0.001	0.000	0.000	0.000	0.000
14	0.025	0.005	0.001	0.000	0.000	0.000	0.000
15	0.025	0.005	0.001	0.000	0.000	0.000	0.000
16	0.025	0.005	0.001	0.000	0.000	0.000	0.000
17	0.025	0.005	0.001	0.000	0.000	0.000	0.000
18	0.025	0.005	0.001	0.000	0.000	0.000	0.000
19	0.025	0.005	0.001	0.000	0.000	0.000	0.000
20	0.025	0.005	0.001	0.000	0.000	0.000	0.000

True Sigma Sei 0.040

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.054	0.025	0.010	0.003	0.001	0.000	0.000
3	0.057	0.025	0.009	0.003	0.001	0.000	0.000
4	0.059	0.025	0.009	0.003	0.001	0.000	0.000
5	0.061	0.025	0.009	0.002	0.001	0.000	0.000
6	0.062	0.025	0.008	0.002	0.001	0.000	0.000
7	0.063	0.025	0.008	0.002	0.000	0.000	0.000
8	0.063	0.025	0.008	0.002	0.000	0.000	0.000
9	0.064	0.025	0.008	0.002	0.000	0.000	0.000
10	0.065	0.025	0.008	0.002	0.000	0.000	0.000
11	0.065	0.025	0.008	0.002	0.000	0.000	0.000
12	0.065	0.025	0.008	0.002	0.000	0.000	0.000
13	0.066	0.025	0.008	0.002	0.000	0.000	0.000
14	0.066	0.025	0.008	0.002	0.000	0.000	0.000
15	0.066	0.025	0.008	0.002	0.000	0.000	0.000
16	0.067	0.025	0.008	0.002	0.000	0.000	0.000
17	0.067	0.025	0.008	0.002	0.000	0.000	0.000
18	0.067	0.025	0.008	0.002	0.000	0.000	0.000
19	0.067	0.025	0.008	0.002	0.000	0.000	0.000
20	0.067	0.025	0.008	0.002	0.000	0.000	0.000

Table 4 - Continued

True Sigma Sei 0.050

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.089	0.050	0.025	0.011	0.004	0.002	0.000
3	0.095	0.052	0.025	0.011	0.004	0.001	0.000
4	0.099	0.053	0.025	0.010	0.004	0.001	0.000
5	0.102	0.054	0.025	0.010	0.004	0.001	0.000
6	0.104	0.054	0.025	0.010	0.004	0.001	0.000
7	0.105	0.055	0.025	0.010	0.004	0.001	0.000
8	0.107	0.055	0.025	0.010	0.003	0.001	0.000
9	0.108	0.055	0.025	0.010	0.003	0.001	0.000
10	0.109	0.056	0.025	0.010	0.003	0.001	0.000
11	0.110	0.056	0.025	0.010	0.003	0.001	0.000
12	0.110	0.056	0.025	0.010	0.003	0.001	0.000
13	0.111	0.056	0.025	0.010	0.003	0.001	0.000
14	0.112	0.056	0.025	0.010	0.003	0.001	0.000
15	0.112	0.056	0.025	0.010	0.003	0.001	0.000
16	0.113	0.057	0.025	0.010	0.003	0.001	0.000
17	0.113	0.057	0.025	0.010	0.003	0.001	0.000
18	0.113	0.057	0.025	0.010	0.003	0.001	0.000
19	0.114	0.057	0.025	0.010	0.003	0.001	0.000
20	0.114	0.057	0.025	0.010	0.003	0.001	0.000

True Sigma Sei 0.060

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.124	0.080	0.047	0.025	0.012	0.006	0.002
3	0.132	0.083	0.048	0.025	0.012	0.005	0.002
4	0.137	0.085	0.048	0.025	0.012	0.005	0.002
5	0.141	0.087	0.049	0.025	0.012	0.005	0.002
6	0.144	0.088	0.049	0.025	0.012	0.005	0.002
7	0.146	0.089	0.049	0.025	0.012	0.005	0.002
8	0.148	0.089	0.049	0.025	0.012	0.005	0.002
9	0.149	0.090	0.050	0.025	0.011	0.005	0.002
10	0.150	0.090	0.050	0.025	0.011	0.005	0.002
11	0.151	0.091	0.050	0.025	0.011	0.005	0.002
12	0.152	0.091	0.050	0.025	0.011	0.005	0.002
13	0.153	0.092	0.050	0.025	0.011	0.005	0.002
14	0.154	0.092	0.050	0.025	0.011	0.005	0.002
15	0.154	0.092	0.050	0.025	0.011	0.005	0.002
16	0.155	0.092	0.050	0.025	0.011	0.005	0.002
17	0.155	0.092	0.050	0.025	0.011	0.005	0.002
18	0.156	0.093	0.050	0.025	0.011	0.005	0.002
19	0.156	0.093	0.050	0.025	0.011	0.005	0.002
20	0.156	0.093	0.050	0.025	0.011	0.005	0.002

Table 4 - Continued

True Sigma Sei 0.070

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.157	0.110	0.072	0.044	0.025	0.013	0.007
3	0.166	0.114	0.074	0.044	0.025	0.013	0.007
4	0.172	0.117	0.075	0.045	0.025	0.013	0.006
5	0.176	0.119	0.076	0.045	0.025	0.013	0.006
6	0.179	0.121	0.076	0.045	0.025	0.013	0.006
7	0.181	0.122	0.077	0.045	0.025	0.013	0.006
8	0.183	0.123	0.077	0.045	0.025	0.013	0.006
9	0.185	0.124	0.078	0.046	0.025	0.013	0.006
10	0.186	0.124	0.078	0.046	0.025	0.013	0.006
11	0.187	0.125	0.078	0.046	0.025	0.013	0.006
12	0.188	0.125	0.078	0.046	0.025	0.013	0.006
13	0.189	0.126	0.079	0.046	0.025	0.013	0.006
14	0.190	0.126	0.079	0.046	0.025	0.013	0.006
15	0.190	0.127	0.079	0.046	0.025	0.013	0.006
16	0.191	0.127	0.079	0.046	0.025	0.013	0.006
17	0.191	0.127	0.079	0.046	0.025	0.013	0.006
18	0.192	0.127	0.079	0.046	0.025	0.013	0.006
19	0.192	0.127	0.079	0.046	0.025	0.013	0.006
20	0.193	0.128	0.079	0.046	0.025	0.013	0.006

True Sigma Sei 0.080

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.186	0.138	0.097	0.065	0.041	0.025	0.014
3	0.196	0.144	0.100	0.066	0.042	0.025	0.014
4	0.202	0.147	0.102	0.067	0.042	0.025	0.014
5	0.206	0.150	0.103	0.068	0.042	0.025	0.014
6	0.209	0.152	0.104	0.068	0.042	0.025	0.014
7	0.212	0.153	0.105	0.068	0.042	0.025	0.014
8	0.214	0.154	0.106	0.069	0.043	0.025	0.014
9	0.215	0.155	0.106	0.069	0.043	0.025	0.014
10	0.217	0.156	0.106	0.069	0.043	0.025	0.014
11	0.218	0.156	0.107	0.069	0.043	0.025	0.014
12	0.219	0.157	0.107	0.069	0.043	0.025	0.014
13	0.220	0.157	0.107	0.069	0.043	0.025	0.014
14	0.220	0.158	0.107	0.070	0.043	0.025	0.014
15	0.221	0.158	0.108	0.070	0.043	0.025	0.014
16	0.222	0.158	0.108	0.070	0.043	0.025	0.014
17	0.222	0.159	0.108	0.070	0.043	0.025	0.014
18	0.223	0.159	0.108	0.070	0.043	0.025	0.014
19	0.223	0.159	0.108	0.070	0.043	0.025	0.014
20	0.223	0.159	0.108	0.070	0.043	0.025	0.014

Table 4 - Continued

True Sigma Sei 0.090

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	0.212	0.165	0.123	0.088	0.060	0.040	0.025
3	0.222	0.171	0.126	0.089	0.061	0.040	0.025
4	0.228	0.175	0.128	0.091	0.061	0.040	0.025
5	0.232	0.177	0.130	0.091	0.062	0.040	0.025
6	0.235	0.179	0.131	0.092	0.062	0.040	0.025
7	0.238	0.181	0.132	0.092	0.062	0.040	0.025
8	0.240	0.182	0.132	0.093	0.062	0.040	0.025
9	0.241	0.183	0.133	0.093	0.063	0.040	0.025
10	0.243	0.184	0.133	0.093	0.063	0.040	0.025
11	0.244	0.184	0.134	0.093	0.063	0.040	0.025
12	0.245	0.185	0.134	0.094	0.063	0.040	0.025
13	0.245	0.185	0.134	0.094	0.063	0.040	0.025
14	0.246	0.186	0.135	0.094	0.063	0.040	0.025
15	0.247	0.186	0.135	0.094	0.063	0.040	0.025
16	0.247	0.186	0.135	0.094	0.063	0.041	0.025
17	0.248	0.187	0.135	0.094	0.063	0.041	0.025
18	0.248	0.187	0.135	0.094	0.063	0.041	0.025
19	0.249	0.187	0.136	0.094	0.063	0.041	0.025
20	0.249	0.187	0.136	0.094	0.063	0.041	0.025

Table 5

Actual Significance Levels for Case II
when True Variances are Unknown

Lambda = 0.025 B = 1
Sigma Cor = 0.040

True Sigma Sei= 0.030

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.025	0.021	0.019	0.017	0.016	0.016	0.015
4	0.025	0.019	0.016	0.014	0.013	0.013	0.012
5	0.025	0.018	0.014	0.012	0.011	0.010	0.010
6	0.025	0.017	0.013	0.011	0.010	0.009	0.008
7	0.025	0.016	0.012	0.010	0.009	0.008	0.007
8	0.025	0.016	0.011	0.009	0.008	0.007	0.006
9	0.025	0.015	0.011	0.008	0.007	0.006	0.006
10	0.025	0.015	0.010	0.008	0.007	0.006	0.005
11	0.025	0.014	0.010	0.007	0.006	0.005	0.005
12	0.025	0.014	0.009	0.007	0.006	0.005	0.004
13	0.025	0.014	0.009	0.007	0.005	0.005	0.004
14	0.025	0.014	0.009	0.007	0.005	0.004	0.004
15	0.025	0.013	0.009	0.006	0.005	0.004	0.004
16	0.025	0.013	0.008	0.006	0.005	0.004	0.004
17	0.025	0.013	0.008	0.006	0.005	0.004	0.003
18	0.025	0.013	0.008	0.006	0.005	0.004	0.003
19	0.025	0.013	0.008	0.006	0.004	0.004	0.003
20	0.025	0.013	0.008	0.006	0.004	0.004	0.003

True Sigma Sei= 0.040

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.030	0.025	0.022	0.021	0.019	0.019	0.018
4	0.032	0.025	0.021	0.019	0.017	0.016	0.016
5	0.034	0.025	0.020	0.018	0.016	0.015	0.014
6	0.036	0.025	0.020	0.017	0.015	0.013	0.013
7	0.037	0.025	0.019	0.016	0.014	0.013	0.012
8	0.038	0.025	0.019	0.015	0.013	0.012	0.011
9	0.039	0.025	0.018	0.015	0.013	0.011	0.010
10	0.039	0.025	0.018	0.014	0.012	0.011	0.010
11	0.040	0.025	0.018	0.014	0.012	0.010	0.010
12	0.041	0.025	0.018	0.014	0.012	0.010	0.009
13	0.041	0.025	0.017	0.014	0.011	0.010	0.009
14	0.042	0.025	0.017	0.013	0.011	0.010	0.009
15	0.042	0.025	0.017	0.013	0.011	0.009	0.009
16	0.042	0.025	0.017	0.013	0.011	0.009	0.008
17	0.043	0.025	0.017	0.013	0.011	0.009	0.008
18	0.043	0.025	0.017	0.013	0.010	0.009	0.008
19	0.043	0.025	0.017	0.013	0.010	0.009	0.008
20	0.043	0.025	0.017	0.013	0.010	0.009	0.008

Table 5 - Continued

True Sigma Sei= 0.050

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.033	0.028	0.025	0.023	0.022	0.021	0.020
4	0.038	0.029	0.025	0.022	0.021	0.020	0.019
5	0.041	0.031	0.025	0.022	0.020	0.018	0.017
6	0.044	0.031	0.025	0.021	0.019	0.018	0.017
7	0.046	0.032	0.025	0.021	0.018	0.017	0.016
8	0.048	0.033	0.025	0.021	0.018	0.016	0.015
9	0.050	0.033	0.025	0.020	0.018	0.016	0.015
10	0.051	0.034	0.025	0.020	0.017	0.016	0.014
11	0.052	0.034	0.025	0.020	0.017	0.015	0.014
12	0.053	0.034	0.025	0.020	0.017	0.015	0.014
13	0.054	0.034	0.025	0.020	0.017	0.015	0.014
14	0.055	0.035	0.025	0.020	0.017	0.015	0.014
15	0.055	0.035	0.025	0.020	0.017	0.015	0.013
16	0.056	0.035	0.025	0.020	0.016	0.015	0.013
17	0.056	0.035	0.025	0.020	0.016	0.014	0.013
18	0.057	0.035	0.025	0.019	0.016	0.014	0.013
19	0.057	0.035	0.025	0.019	0.016	0.014	0.013
20	0.058	0.036	0.025	0.019	0.016	0.014	0.013

True Sigma Sei= 0.060

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.036	0.030	0.027	0.025	0.024	0.023	0.022
4	0.042	0.033	0.028	0.025	0.023	0.022	0.021
5	0.046	0.035	0.029	0.025	0.023	0.021	0.020
6	0.050	0.036	0.029	0.025	0.022	0.021	0.020
7	0.053	0.038	0.030	0.025	0.022	0.020	0.019
8	0.056	0.039	0.030	0.025	0.022	0.020	0.019
9	0.058	0.040	0.030	0.025	0.022	0.020	0.018
10	0.059	0.040	0.030	0.025	0.022	0.020	0.018
11	0.061	0.041	0.031	0.025	0.022	0.019	0.018
12	0.062	0.041	0.031	0.025	0.022	0.019	0.018
13	0.063	0.042	0.031	0.025	0.021	0.019	0.018
14	0.064	0.042	0.031	0.025	0.021	0.019	0.018
15	0.065	0.043	0.031	0.025	0.021	0.019	0.017
16	0.066	0.043	0.031	0.025	0.021	0.019	0.017
17	0.066	0.043	0.031	0.025	0.021	0.019	0.017
18	0.067	0.043	0.031	0.025	0.021	0.019	0.017
19	0.068	0.044	0.032	0.025	0.021	0.019	0.017
20	0.068	0.044	0.032	0.025	0.021	0.019	0.017

Table 5 - Continued

True Sigma Sei= 0.070

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.038	0.032	0.028	0.026	0.025	0.024	0.023
4	0.045	0.035	0.030	0.027	0.025	0.024	0.023
5	0.050	0.038	0.031	0.027	0.025	0.023	0.022
6	0.055	0.040	0.032	0.028	0.025	0.023	0.022
7	0.058	0.042	0.033	0.028	0.025	0.023	0.022
8	0.061	0.043	0.034	0.028	0.025	0.023	0.021
9	0.064	0.044	0.034	0.028	0.025	0.023	0.021
10	0.066	0.045	0.035	0.029	0.025	0.023	0.021
11	0.067	0.046	0.035	0.029	0.025	0.023	0.021
12	0.069	0.047	0.035	0.029	0.025	0.023	0.021
13	0.070	0.047	0.035	0.029	0.025	0.022	0.021
14	0.071	0.048	0.036	0.029	0.025	0.022	0.021
15	0.072	0.048	0.036	0.029	0.025	0.022	0.021
16	0.073	0.049	0.036	0.029	0.025	0.022	0.021
17	0.074	0.049	0.036	0.029	0.025	0.022	0.020
18	0.074	0.049	0.036	0.029	0.025	0.022	0.020
19	0.075	0.050	0.037	0.029	0.025	0.022	0.020
20	0.076	0.050	0.037	0.029	0.025	0.022	0.020

True Sigma Sei= 0.080

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.039	0.033	0.030	0.027	0.026	0.025	0.024
4	0.047	0.037	0.032	0.029	0.026	0.025	0.024
5	0.053	0.040	0.033	0.029	0.027	0.025	0.024
6	0.058	0.043	0.035	0.030	0.027	0.025	0.024
7	0.062	0.045	0.036	0.030	0.027	0.025	0.024
8	0.065	0.046	0.037	0.031	0.027	0.025	0.023
9	0.068	0.048	0.037	0.031	0.027	0.025	0.023
10	0.070	0.049	0.038	0.031	0.028	0.025	0.023
11	0.072	0.050	0.038	0.032	0.028	0.025	0.023
12	0.074	0.051	0.039	0.032	0.028	0.025	0.023
13	0.075	0.051	0.039	0.032	0.028	0.025	0.023
14	0.076	0.052	0.039	0.032	0.028	0.025	0.023
15	0.077	0.052	0.039	0.032	0.028	0.025	0.023
16	0.078	0.053	0.040	0.032	0.028	0.025	0.023
17	0.079	0.053	0.040	0.032	0.028	0.025	0.023
18	0.080	0.054	0.040	0.033	0.028	0.025	0.023
19	0.081	0.054	0.040	0.033	0.028	0.025	0.023
20	0.081	0.054	0.040	0.033	0.028	0.025	0.023

Table 5 - Continued

True Sigma Sei= 0.090

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	0.040	0.034	0.030	0.028	0.027	0.026	0.025
4	0.049	0.039	0.033	0.030	0.027	0.026	0.025
5	0.055	0.042	0.035	0.031	0.028	0.026	0.025
6	0.061	0.045	0.037	0.032	0.028	0.026	0.025
7	0.065	0.047	0.038	0.032	0.029	0.027	0.025
8	0.068	0.049	0.039	0.033	0.029	0.027	0.025
9	0.071	0.050	0.039	0.033	0.029	0.027	0.025
10	0.074	0.052	0.040	0.034	0.029	0.027	0.025
11	0.076	0.053	0.041	0.034	0.030	0.027	0.025
12	0.077	0.054	0.041	0.034	0.030	0.027	0.025
13	0.079	0.054	0.042	0.034	0.030	0.027	0.025
14	0.080	0.055	0.042	0.034	0.030	0.027	0.025
15	0.08[illegible]	0.056	0.042	0.035	0.030	0.027	0.025
16	0.082	0.056	0.042	0.035	0.030	0.027	0.025
17	0.083	0.057	0.043	0.035	0.030	0.027	0.025
18	0.084	0.057	0.043	0.035	0.030	0.027	0.025
19	0.085	0.057	0.043	0.035	0.030	0.027	0.025
20	0.085	0.058	0.043	0.035	0.030	0.027	0.025

Table 6

F-Numbers for Case II
when True Variances are Unknown

Lambda = 0.025 B = 1
Sigma Cor = 0.040

True Sigma Sei= 0.030

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.43	1.48	1.52	1.54	1.56	1.58	1.59
4	1.30	1.33	1.36	1.38	1.39	1.40	1.41
5	1.25	1.28	1.31	1.32	1.34	1.35	1.35
6	1.22	1.26	1.28	1.29	1.31	1.32	1.32
7	1.21	1.24	1.26	1.28	1.29	1.30	1.30
8	1.20	1.23	1.25	1.27	1.28	1.28	1.29
9	1.19	1.22	1.24	1.26	1.27	1.28	1.28
10	1.19	1.22	1.24	1.25	1.26	1.27	1.28
11	1.18	1.21	1.23	1.25	1.26	1.26	1.27
12	1.18	1.21	1.23	1.24	1.25	1.26	1.27
13	1.18	1.20	1.22	1.24	1.25	1.26	1.26
14	1.17	1.20	1.22	1.24	1.25	1.25	1.26
15	1.17	1.20	1.22	1.23	1.24	1.25	1.26
16	1.17	1.20	1.22	1.23	1.24	1.25	1.26
17	1.17	1.20	1.22	1.23	1.24	1.25	1.25
18	1.17	1.19	1.21	1.23	1.24	1.25	1.25
19	1.17	1.19	1.21	1.23	1.24	1.25	1.25
20	1.16	1.19	1.21	1.23	1.24	1.24	1.25

True Sigma Sei= 0.040

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.50	1.56	1.60	1.63	1.66	1.68	1.69
4	1.34	1.38	1.42	1.44	1.46	1.47	1.48
5	1.29	1.32	1.35	1.37	1.39	1.40	1.41
6	1.26	1.29	1.32	1.34	1.35	1.36	1.37
7	1.24	1.27	1.30	1.32	1.33	1.34	1.35
8	1.23	1.26	1.29	1.31	1.32	1.33	1.34
9	1.22	1.25	1.28	1.30	1.31	1.32	1.32
10	1.21	1.25	1.27	1.29	1.30	1.31	1.32
11	1.21	1.24	1.27	1.28	1.30	1.30	1.31
12	1.20	1.24	1.26	1.28	1.29	1.30	1.31
13	1.20	1.23	1.26	1.27	1.29	1.30	1.30
14	1.20	1.23	1.25	1.27	1.28	1.29	1.30
15	1.20	1.23	1.25	1.27	1.28	1.29	1.30
16	1.19	1.23	1.25	1.27	1.28	1.29	1.29
17	1.19	1.22	1.25	1.26	1.28	1.29	1.29
18	1.19	1.22	1.25	1.26	1.27	1.28	1.29
19	1.19	1.22	1.24	1.26	1.27	1.28	1.29
20	1.19	1.22	1.24	1.26	1.27	1.28	1.29

Table 6 - Continued

True Sigma Sei= 0.050

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.58	1.65	1.70	1.74	1.77	1.79	1.81
4	1.39	1.44	1.48	1.51	1.53	1.55	1.56
5	1.33	1.37	1.41	1.43	1.45	1.46	1.47
6	1.30	1.34	1.37	1.39	1.41	1.42	1.43
7	1.28	1.32	1.35	1.37	1.38	1.40	1.40
8	1.26	1.30	1.33	1.35	1.37	1.38	1.39
9	1.25	1.29	1.32	1.34	1.36	1.37	1.37
10	1.24	1.28	1.31	1.33	1.35	1.36	1.37
11	1.24	1.28	1.31	1.33	1.34	1.35	1.36
12	1.23	1.27	1.30	1.32	1.33	1.35	1.35
13	1.23	1.27	1.30	1.32	1.33	1.34	1.35
14	1.23	1.27	1.29	1.31	1.33	1.34	1.34
15	1.22	1.26	1.29	1.31	1.32	1.33	1.34
16	1.22	1.26	1.29	1.31	1.32	1.33	1.34
17	1.22	1.26	1.28	1.30	1.32	1.33	1.34
18	1.22	1.26	1.28	1.30	1.32	1.33	1.33
19	1.22	1.25	1.28	1.30	1.31	1.32	1.33
20	1.21	1.25	1.28	1.30	1.31	1.32	1.33

True Sigma Sei= 0.060

	Estimated Sigma Sei						
k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.67	1.76	1.82	1.87	1.90	1.93	1.95
4	1.45	1.51	1.56	1.59	1.62	1.63	1.65
5	1.38	1.43	1.47	1.50	1.52	1.53	1.54
6	1.34	1.39	1.42	1.45	1.47	1.48	1.50
7	1.32	1.36	1.40	1.42	1.44	1.45	1.47
8	1.30	1.35	1.38	1.40	1.42	1.44	1.45
9	1.29	1.33	1.37	1.39	1.41	1.42	1.43
10	1.28	1.32	1.36	1.38	1.40	1.41	1.42
11	1.27	1.32	1.35	1.37	1.39	1.40	1.41
12	1.27	1.31	1.34	1.37	1.38	1.40	1.41
13	1.26	1.31	1.34	1.36	1.38	1.39	1.40
14	1.26	1.30	1.34	1.36	1.37	1.39	1.40
15	1.26	1.30	1.33	1.35	1.37	1.38	1.39
16	1.25	1.30	1.33	1.35	1.37	1.38	1.39
17	1.25	1.29	1.33	1.35	1.37	1.38	1.39
18	1.25	1.29	1.32	1.35	1.36	1.37	1.38
19	1.25	1.29	1.32	1.34	1.36	1.37	1.38
20	1.24	1.29	1.32	1.34	1.36	1.37	1.38

Table 6 - Continued

True Sigma Sei= 0.070

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.78	1.88	1.96	2.01	2.06	2.09	2.11
4	1.52	1.59	1.64	1.68	1.71	1.73	1.75
5	1.43	1.49	1.54	1.57	1.59	1.61	1.63
6	1.39	1.44	1.49	1.52	1.54	1.56	1.57
7	1.36	1.41	1.45	1.48	1.51	1.52	1.53
8	1.34	1.39	1.43	1.46	1.48	1.50	1.51
9	1.33	1.38	1.42	1.45	1.47	1.48	1.49
10	1.32	1.37	1.41	1.44	1.45	1.47	1.48
11	1.31	1.36	1.40	1.43	1.45	1.46	1.47
12	1.30	1.35	1.39	1.42	1.44	1.45	1.46
13	1.30	1.35	1.39	1.41	1.43	1.45	1.46
14	1.29	1.34	1.38	1.41	1.43	1.44	1.45
15	1.29	1.34	1.38	1.40	1.42	1.44	1.45
16	1.29	1.34	1.37	1.40	1.42	1.43	1.44
17	1.28	1.33	1.37	1.40	1.42	1.43	1.44
18	1.28	1.33	1.37	1.39	1.41	1.43	1.44
19	1.28	1.33	1.37	1.39	1.41	1.42	1.44
20	1.28	1.33	1.36	1.39	1.41	1.42	1.43

True Sigma Sei= 0.080

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	1.89	2.01	2.10	2.17	2.22	2.26	2.29
4	1.59	1.67	1.73	1.78	1.81	1.84	1.86
5	1.49	1.56	1.61	1.65	1.68	1.70	1.72
6	1.44	1.50	1.55	1.59	1.61	1.63	1.65
7	1.40	1.47	1.52	1.55	1.57	1.59	1.61
8	1.38	1.45	1.49	1.52	1.55	1.57	1.58
9	1.37	1.43	1.47	1.51	1.53	1.55	1.56
10	1.36	1.42	1.46	1.49	1.52	1.53	1.55
11	1.35	1.41	1.45	1.48	1.51	1.52	1.53
12	1.34	1.40	1.44	1.47	1.50	1.51	1.53
13	1.34	1.39	1.44	1.47	1.49	1.51	1.52
14	1.33	1.39	1.43	1.46	1.48	1.50	1.51
15	1.33	1.38	1.43	1.46	1.48	1.50	1.51
16	1.32	1.38	1.42	1.45	1.47	1.49	1.50
17	1.32	1.38	1.42	1.45	1.47	1.49	1.50
18	1.32	1.37	1.42	1.45	1.47	1.48	1.50
19	1.31	1.37	1.41	1.44	1.47	1.48	1.49
20	1.31	1.37	1.41	1.44	1.46	1.48	1.49

Table 6 - Continued

True Sigma Sei= 0.090

Estimated Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3	2.02	2.16	2.27	2.35	2.41	2.46	2.49
4	1.67	1.76	1.83	1.89	1.92	1.95	1.98
5	1.55	1.63	1.69	1.74	1.77	1.79	1.81
6	1.49	1.57	1.62	1.66	1.69	1.72	1.73
7	1.45	1.53	1.58	1.62	1.65	1.67	1.68
8	1.43	1.50	1.55	1.59	1.62	1.64	1.65
9	1.41	1.48	1.53	1.57	1.60	1.62	1.63
10	1.40	1.47	1.52	1.55	1.58	1.60	1.62
11	1.39	1.46	1.51	1.54	1.57	1.59	1.60
12	1.38	1.45	1.50	1.53	1.56	1.58	1.59
13	1.37	1.44	1.49	1.53	1.55	1.57	1.58
14	1.37	1.44	1.48	1.52	1.54	1.56	1.58
15	1.36	1.43	1.48	1.51	1.54	1.56	1.57
16	1.36	1.43	1.47	1.51	1.53	1.55	1.57
17	1.36	1.42	1.47	1.50	1.53	1.55	1.56
18	1.35	1.42	1.47	1.50	1.53	1.54	1.56
19	1.35	1.42	1.46	1.50	1.52	1.54	1.55
20	1.35	1.41	1.46	1.50	1.52	1.54	1.55

Table 7

F-Numbers for Modified Case II in (2.23)

LAMBDA = .025
B = 1

True Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	3.36	3.94	4.72	5.74	7.05	8.74	10.88
3	1.62	1.72	1.85	2.00	2.17	2.37	2.58
4	1.44	1.52	1.60	1.70	1.81	1.93	2.07
5	1.38	1.44	1.51	1.60	1.69	1.79	1.90
6	1.35	1.41	1.47	1.54	1.63	1.72	1.81
7	1.33	1.38	1.44	1.51	1.59	1.67	1.76
8	1.32	1.37	1.43	1.49	1.56	1.64	1.73
9	1.31	1.36	1.41	1.48	1.55	1.62	1.70
10	1.30	1.35	1.40	1.47	1.53	1.61	1.69
11	1.30	1.34	1.40	1.46	1.52	1.59	1.67
12	1.29	1.34	1.39	1.45	1.51	1.58	1.66
13	1.29	1.33	1.39	1.44	1.51	1.58	1.65
14.	1.29	1.33	1.38	1.44	1.50	1.57	1.64
15	1.28	1.33	1.38	1.43	1.50	1.56	1.64
16	1.28	1.32	1.37	1.43	1.49	1.56	1.63
17	1.28	1.32	1.37	1.43	1.49	1.56	1.63
18	1.28	1.32	1.37	1.43	1.49	1.55	1.62
19	1.28	1.32	1.37	1.42	1.48	1.55	1.62
20	1.28	1.32	1.37	1.42	1.48	1.55	1.62

Table 8

Actual Significance Levels for
Modified Case II Test in (2.23)

LAMBDA = .025
B = 1

True Sigma Sei

k	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2	.019	.020	.022	.022	.023	.023	.024
3	.013	.016	.018	.020	.021	.021	.022
4	.010	.013	.016	.018	.019	.020	.021
5	.008	.011	.014	.016	.018	.019	.020
6	.006	.010	.013	.015	.017	.018	.020
7	.005	.009	.012	.014	.016	.018	.019
8	.004	.008	.011	.014	.016	.018	.019
9	.004	.007	.011	.013	.015	.017	.019
10	.003	.007	.010	.013	.015	.017	.018
11	.003	.006	.010	.013	.015	.017	.018
12	.003	.006	.009	.012	.015	.016	.018
13	.003	.006	.009	.012	.014	.016	.018
14	.002	.006	.009	.012	.014	.016	.018
15	.002	.005	.009	.012	.014	.016	.018
16	.002	.005	.009	.012	.014	.016	.017
17	.002	.005	.008	.011	.014	.016	.017
18	.002	.005	.008	.011	.014	.016	.017
19	.002	.005	.008	.011	.014	.016	.017
20	.002	.005	.008	.011	.014	.016	.017

Table 9 . Bias and $\sqrt{MSE}$ for Estimating σ_{sei}* using Mixture-of-Normals Approach

Number of Components

	2		3		4	
	bias	$\sqrt{MSE}$	bias	$\sqrt{MSE}$	bias	$\sqrt{MSE}$
1.5σ	.006	.018	.016	.029	.018	.035
2σ	.006	.023	.011	.029	.017	.029
2.5σ	.001	.022	.008	.024	.017	.032
5σ	−.005	.010	−.004	.008	−.003	.007

* True σ_{sei} = .06

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